# HSC Stage 6 2026 Mathematics Extension II Syllabus Comments

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This is a document containing my observations, notes and opinions regarding the HSC Mathematics Extension II syllabus for implementation in 2026.

I have attempted to be impartial in the main dotpoints listed and placed my (sometimes critical) opinions in the footnotes.

## 1 Year 12 Mathematics Extension II

#### 1.1 The nature of proof

- More clarity in the language and notation of proof. The logical symbols for "and" ∧, "or" ∨ and negation ¬ (or ~) are now in the syllabus.
- The negation rules (De Morgan's Laws)  $\neg (P \land Q) = \neg P \lor \neg Q$  and  $\neg (P \lor Q) = \neg P \land \neg Q$  are explicitly mentioned in the syllabus, as well as the result for the negation of implication  $\neg (P \Rightarrow Q) = P \land \neg Q$ .<sup>1</sup>
- Teaching considerations states that it may be beneficial for some students to extend their knowledge to analyse further logic statements using tools such as truth tables, but this is not essential content. This statement should set the precedent that all teaching considerations that are labeled inessential are *not* assessable, but are there to provide enrichment to the learning experience of students.
- The teaching considerations make note of how to negate quantified statements (with a peculiar example about fluffy cats), but is not explicitly mentioned in the syllabus dotpoints. It is interesting to see that the dotpoint about quantifiers was moved from the bottom of this subsection in the draft to right before the dotpoint about negations in the final version. Hence, it is reasonable to assume that negation of quantified statements is assessable.

The negation of quantified statements are:  $\neg(\forall x : P(x)) = \exists x : \neg P(x)$  and  $\neg(\exists x : P(x)) = \forall x : \neg P(x)$ .

<sup>&</sup>lt;sup>1</sup>This levels the playing field for students. In the 2017 syllabus, it was not immediately clear that these concepts should be covered. Some textbooks (and by extension some schools) did not teach this, but these concepts have been assessed in the HSC papers.

- Techniques of proof unchanged from 2017 syllabus. Vague language such as "simple" removed. The dotpoint about proving results involving integers does not imply all proof questions must be about integers. For example, it should not preclude proving results about irrationality of numbers as prescribed in the 2017 syllabus such as  $\sqrt{2}$  or  $\log_2 5$ , as they fall under the technique of proof by contradiction. Rather, this dotpoint in tandem with the example given is about using basic integer number theoretical facts such as odd/even properties or divisibility facts about integers to substantiate a proof.
- Proofs of inequalities given its own subsection.
- There is a lengthy dotpoint about manipulating a > b (3rd from the top of the Proof of Inequalities subsection).<sup>2</sup>
- Triangle inequality and AM-GM inequality retained.
- The Making Connections table in the Further Work With Vectors section states that the Cauchy-Schwarz inequality for vectors in two or three dimensions immediately gives the result for real numbers of two or three variables respectively. That is,

$$(a_1b_1 + a_2b_2 + a_3b_3)^2 \le (a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2).$$

Note that this inequality has already been assessed in 2021 Question 16 (ii).

- The dotpoint about proving inequalities involving geometry is new but lacks an example.  $^3$
- The squeeze theorem is now in the syllabus. The Making Connections table in this section states that the squeeze theorem requires an understanding of limits and can be related to differentiation by first principles.<sup>4</sup>
- Proving inequalities using graphical or calculus techniques needs a better example than just a statement about considering where functions are increasing or decreasing.

<sup>&</sup>lt;sup>2</sup>This entire dotpoint can be cleaned up more concisely as follows. Given a > b and f is an increasing function, then f(a) > f(b). If f is a decreasing function, then f(a) < f(b) - then put the results given in the syllabus in an example. However, this invokes the teaching consideration about extending students to learn the definition of increasing/decreasing on an interval in the Mathematics Advanced syllabus in the Introduction to Differentiation section.

 $<sup>^3\</sup>mathrm{Some}$  clarity and an example provided from NESA on the scope of this dotpoint would be helpful.

<sup>&</sup>lt;sup>4</sup>There is no definition of limits, not even an informal one, in the Mathematics Advanced syllabus and the dotpoint on computing limits in the draft did not make the cut. I will assume that in Extension II, covering limits a little bit more in detail will be beneficial for students before covering this dotpoint - what a great chance to discuss nested quantifiers and the epsilon-delta definition after having done the language of proof!

Furthermore, this dotpoint about the squeeze theorem will allow teachers to prove the derivatives of trigonometric functions by first principles, which relies on the fundamental limit:  $\lim_{x\to 0} \frac{\sin x}{x} = 1$ . Perhaps this is also an example of proving inequalities involving geometry?

I personally think that with the removal of basic limit calculations from the syllabus, this dotpoint is out of place.

• Further proof by mathematical induction is unchanged from the 2017 syllabus. Note that this is the first occurrence of the term "recurrence relation" in the progression of the Mathematics calculus course syllabuses.

### **1.2** Further work with vectors

- Compared to Extension I, the focus is no longer on extending vectors to 3D (this is now an Extension I concept), but on vector equations of lines and curves, and using vectors to solve geometry problems. The 2026 syllabus expands upon the dotpoints previously under V1.3: Vectors and vector equations of lines, and correctly fixes this title to include curves.
- The use of  $\mathbf{r} = \mathbf{p} + \lambda(\mathbf{q} \mathbf{p})$  (equivalently  $\mathbf{r} = \lambda \mathbf{q} + (1 \lambda)\mathbf{p}$ )) to give a vector equation for the straight line through two points is explicitly mentioned in the syllabus.
- In the 2017 syllabus, the assessable questions were implied to be restricted to intersecting lines in 3D. There is now mention of skew lines in the 2026 syllabus and thus this restriction of questions is lifted.
- This section studies curves and their vector equations.<sup>5</sup>
- Vector equations of circles and spheres given a lot of detail and clarification.
- The dotpoint about determining the Cartesian equation of a curve in 2D given a vector equation and graphing the curve naturally links to parametric equations.
- $\bullet$  The algebraic properties of the scalar (dot) product is explored under the subsection Vectors and Geometry.<sup>6</sup>
- Cauchy-Schwarz inequality now in syllabus.<sup>7</sup>
- The teaching considerations gives a comprehensive list on what geometric proofs can be done with vectors. Teachers should refer to this list when designing tasks.

# **1.3** Introduction to complex numbers

• The 2017 syllabus split complex numbers into two sections: MEX-N1 and MEX-N2. This is now consolidated under one section in the 2026 syllabus.

<sup>&</sup>lt;sup>5</sup>Get ready for more mad spirals!

<sup>&</sup>lt;sup>6</sup>Specifically bilinearity.

<sup>&</sup>lt;sup>7</sup>This addition may seem strange at first, but actually is one that levels the playing field. Take for example Question 16 of the 2021 paper - students who knew of this identity before entering the examination were at an advantage compared to those who didn't. This inequality makes for a very rich discussion and investigation.

- Euler form  $e^{ix} = \cos x + i \sin x$  removed from the syllabus.<sup>8</sup>
- Classification of numbers as Natural N, Integers Z, Rational Numbers Q, Real Numbers ℝ or Complex Numbers C is now in the course.<sup>9</sup>
- The 2017 syllabus had a technical error in defining  $\arg z$  without specifying the importance of the angle being located in the correct quadrant. The 2026 syllabus has now fixed this error.
- The rest of this section follows the classic path of complex number education addition, subtraction, multiplication, division, De Moivre's Theorem, roots of unity and sketching lines, curves and regions.<sup>10</sup>
- Teaching considerations uses  $cis(\theta)$  notation.
- Teaching considerations suggest enriching students' learning with the fundamental theorem of algebra, but this is inessential content.

#### **1.4** Further integration

- The product of trigonometric functions expressed as sums and differences has been moved from Extension I to Extension II in this section and therefore also the technique of integrating products of trigonometric functions.
- *t*-results moved from Extension I to Extension II. It is heavily implied by the sequencing and title of this section that they are to be used for the purpose of computing certain classes of integrals by *t*-result substitution.
- The rest of this section is largely unchanged.
- The teaching considerations draw attention to the use of complex numbers to solve partial fraction decomposition problems.
- The teaching considerations mention the old syllabus formulas to benefit students' learning for  $\int \frac{dx}{\sqrt{x^2 \pm a^2}}$ . Interestingly, although the recommendation is that students are not expected to recognise and use these results, they have not been labeled as inessential content.

<sup>&</sup>lt;sup>8</sup>The rationale for this is that its use case was not much different from  $\cos x + i \sin x$  anyway, but rather it created some problems whereby students (and teachers alike) naively misunderstood the behaviour of complex valued functions, such as the complex logarithm, complex trigonometric functions, and complex valued differential/integral calculus. These ideas are best reserved for a more qualified and careful treatment at university in a Complex Analysis course. I guess you can say that NESA cut this branch.

<sup>&</sup>lt;sup>9</sup>Teaching considerations acknowledge that the inclusion of  $0 \in \mathbb{N}$  is not universally agreed upon. Therefore, this should not be assessed. We can opt for  $\mathbb{Z}^+$  and  $\mathbb{Z}^+ \cup \{0\}$  notation. Nowadays, I've come to personally prefer the notation  $\mathbb{Z}_{>0}$  and  $\mathbb{Z}_{\geq 0}$ .

I would recommend anyone to read up on Peano's Axioms and Von Neumann's ordinals before feeling too strongly about this topic. One can find a reference googling "Set-theoretic definition of natural numbers" and finding the associated Wikipedia page on this.

<sup>&</sup>lt;sup>10</sup>Where have all the examples for these dotpoints gone?

# 1.5 Applications of calculus to mechanics

- More clarity included in this section compared to the 2017 syllabus. The mechanics problems to be considered have not changed.
- Pulley systems and inclined planes are now explicitly mentioned in the syllabus, with reference to the skill of resolving forces into components.
- Newton's law  $\sum F = m\ddot{x}$  now includes the summation symbol to correctly remind readers that all forces must be considered.
- Resisted projectile motion explicitly specifies the magnitude of the resistive force is proportional to the speed (and not the second power of speed).
- The derivation of the Cartesian equation for non-resisted projectile motion has been appropriately moved from Extension II to Extension I.
- Teaching considerations state that essential syllabus content is restricted to problems involving constant mass.